TRIGONOMETRY

INTRODUCTION AND ANALYSIS

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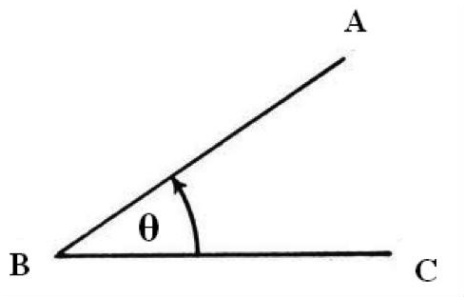
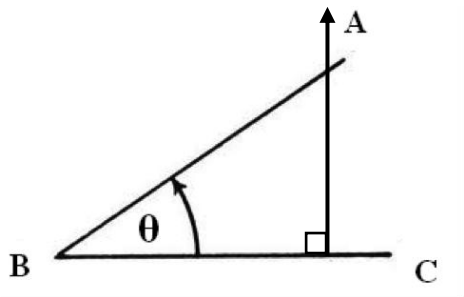
Trigonometry stands for triangle measurement. However, trigonometry is not limited to triangles at all, in fact, it focuses more on angles and using triangles to as an aid to understand angles. Generally, there are two basic approaches to learning trigonometry— the right triangle approach and the unit circle approach. Here, we will cover both topics. Anyhow, all trigonometry concepts are centered around angles anyway, so let us start with an angle.



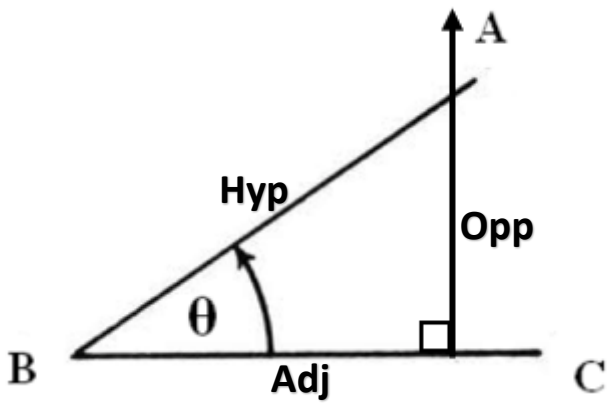
θ or ϑ (theta), is the Greek letter used commonly and by convention to denote an angle in trigonometry. Now let us create an imaginary triangle using a line to describe the trigonometric functions of the angle, θ.



Remember, that the angle formed must be a right angle or 90° degrees, forming a right-angled triangle. You can think of a paper cutter slicing the angle perpendicularly.

We know that the side opposite to the right angle is the hypotenuse. But in trigonometry the other two sides are also named, opposite and adjacent, in relation to angle θ. The side opposite to angle θ, is opp. or opposite and the side that forms

one of the rays of θ, but is not the hypotenuse, i.e. adjacent to θ, is called adj. or adjacent.



Trigonometry focuses on ratios and, the six trigonometric functions are simply the ratios of the different sides of this imaginary right angled triangle formed from θ. Keep in mind, the trigonometric formulas given below only apply to right-angled triangles. Trigonometric functions only deal with angles; we are using right-angled triangles to help better understand. Since there are three sides (Hyp., Adj. and Opp.), there are six possible pairs of ratios that can be formed. Likewise, likewise, the six trigonometric functions ARE these ratios. The six trig. function functions are —

Sine = Opp/Hyp	Cosecant = Hyp/Opp
Cosine = Adj/Hyp	Cosine = Hyp/Adj
Tangent = Opp/Adj	Cotangent = Adj/Opp

There are three main functions- Sine, denoted by sinθ, Cosine, denoted by cosθ, and Tangent, denoted by tanθ. The other functions are simply the inverse of the rest — Cosecant, denoted by cscθ, Secant, denoted by secθ, and Cotangent, denoted by cotθ. They can also be called the 'trigonometric ratios'.

A popular mnemonic for memorising these functions is "SOH-CAH-TOA".

SOH	CAH	TOA
Sine = **O**pp/**H**yp	**C**osine = **A**dj/**H**yp	**T**angent = **O**pp/**A**dj

You can memorise the rest by remembering that there is always one "co" in each pair.

Sine x Cosecant

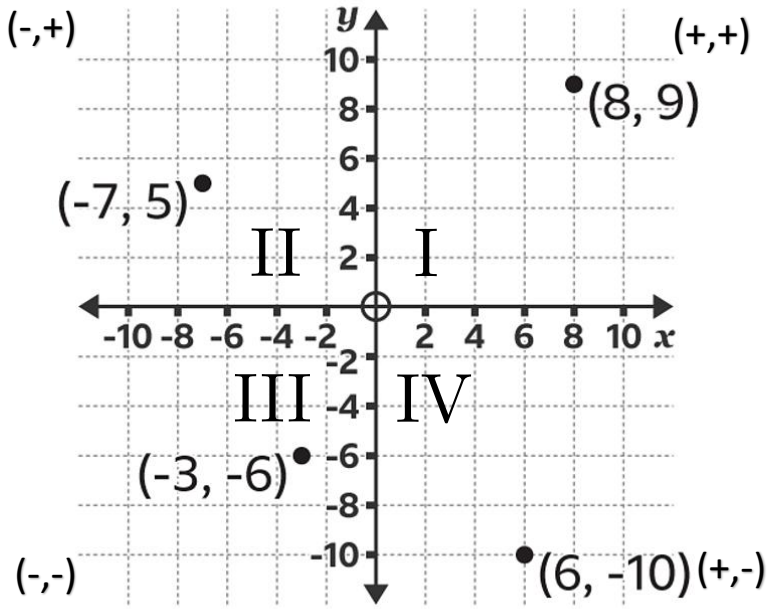
Cosine x Secant

Tangent x Cotangent

While writing and doing trigonometry, there are some key points to remember –

- Trig functions are properties of angles, not triangles.
- Trig functions are unitless.
- Trig functions are written and read as sinθ not sin(θ).
- Squared trig functions are written and read as $\sin^2 0$ and "Sine squared theta" not $(\sin\theta)^2$.

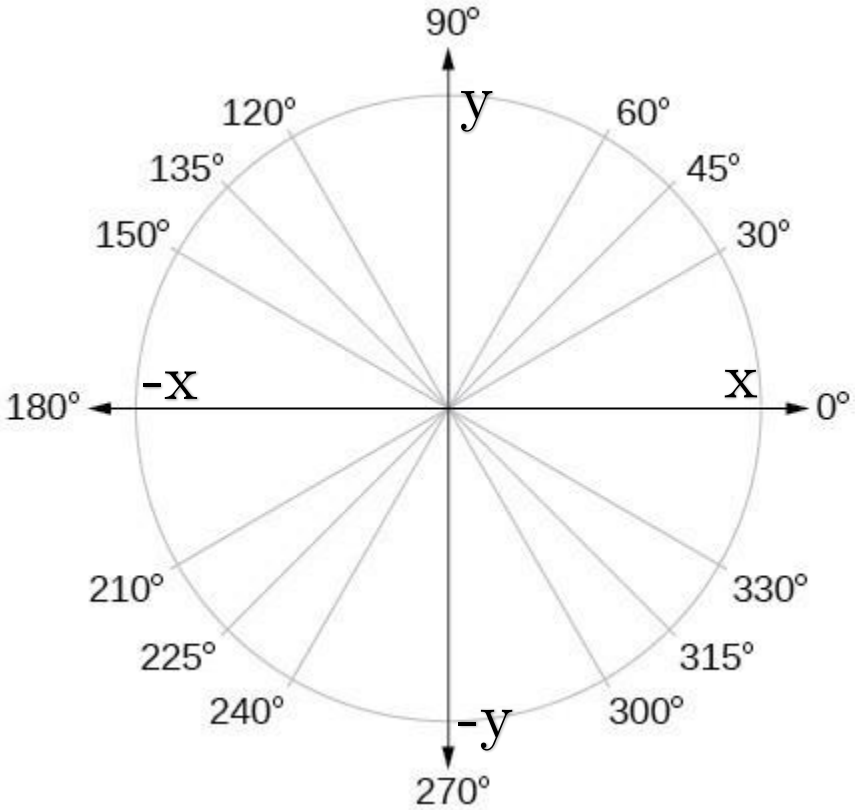
Now we go on the topic of Cartesian coordinates and Quadrants. Imagine a plane with a perpendicular x and y axes. By convention, the x axis is drawn horizontally and the y axis is drawn vertically.



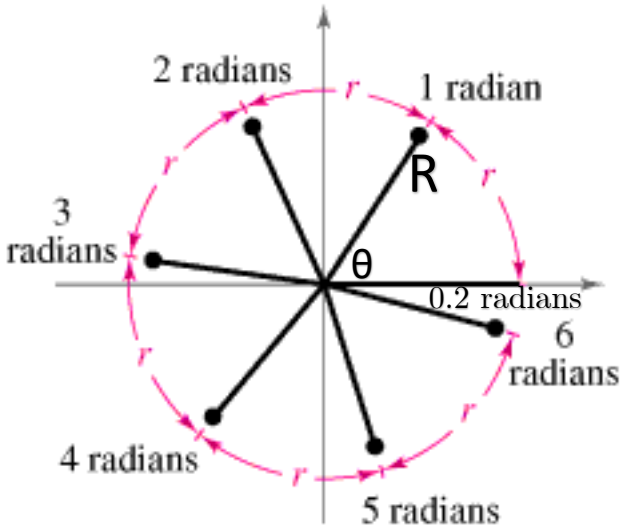
The point where the x and y axis intersect is called the origin. The x and y coordinates start from zero at the origin and go positive to the up and right

directions respectively, and negative to the left and down directions. The plane is divided into four Quadrants — I, II, III, IV. By convention, we mark coordinates as (x, y), for example the coordinate (-7, 5) would be in the II Quadrant. Now that we are familiar with a plane, let us understand how angles are measured in degrees and radians in the context of a circle.

You are probably familiar with expressing angles or measuring them in degrees, but what about radians? Below is a circle with dots at every 30° interval and 45° interval. There are infinite number of angles that can be measured but these are the important ones. Remember that the angles are measured starting from 0° at the positive x axis.

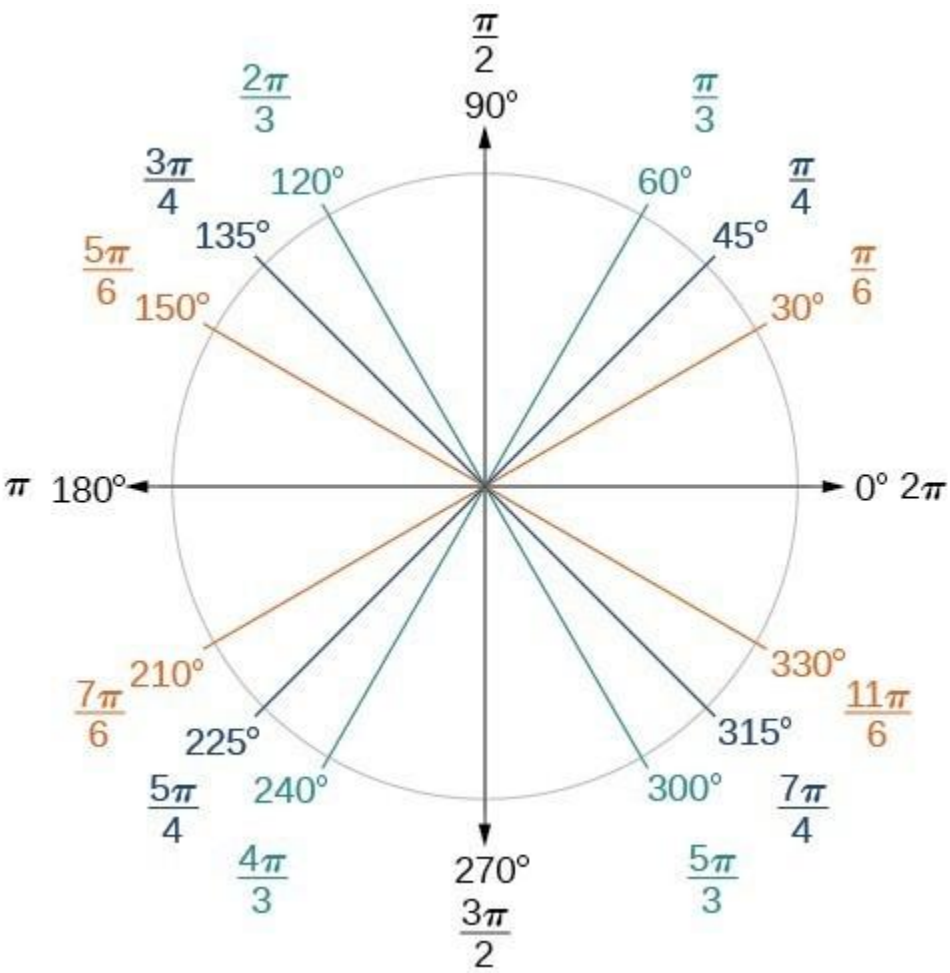


Although using degrees is common, scientists, mathematicians and engineers often use radians as a unit of angle measurement. Imagine a circle with radius of R. Now take R length of the circumference. An angle that intersects those two points on the circumference with its vertex at the centre of the circle, is has the value of 1 radian.



Circumference = 2πR
Radius = R
No. of Radians = 2πR/R
=2π
≈ 6.2

Since the circumference of a circle is 2πR and the length of the radian is R, the no. of radians in one circle is 2π. Let us express now, the degrees in terms of radians.



Since we know that there are 2π radians, half of the circle (180°) must be π radians. Although, it may look unintuitive at first, if we understand that n steps of 30° is $\frac{n\pi}{6}$

because $2\pi \div \frac{360}{30} = \frac{\pi}{6}$ and n steps of 45° is $\frac{n\pi}{4}$ because $2\pi \div \frac{45}{360} = \frac{\pi}{4}$ then conversion becomes simple, really. Here is are some tables to clear things up—

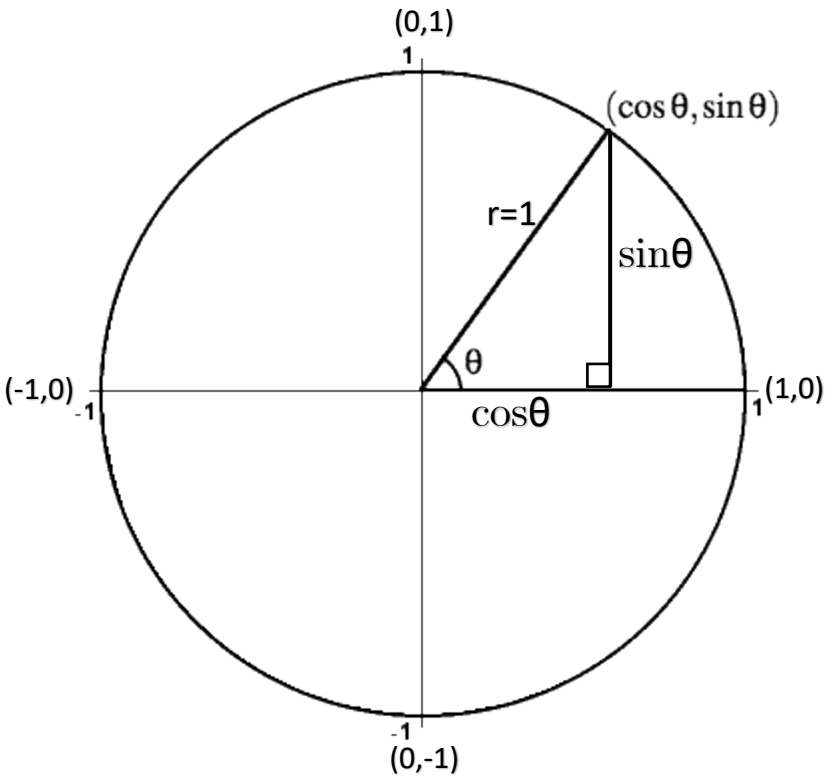
Multiple of 30°	Radians	Simplified
0°	0/6	0
30°	π/6	π/6
60°	2π/6	π/3
90°	3π/6	π/2
120°	4π/6	2π/3
150°	5π/6	5π/6
180°	6π/6	π
210°	7π/6	7π/6
240°	8π/6	4π/3
270°	9π/6	3π/2
300°	10π/6	5π/3
330°	11π/6	11π/6
360°	12π/6	2π

Multiple of 45°	Radians	Simplified
0°	0/4	0
45°	π/4	π/4
90°	2π/4	π/2
135°	3π/4	3π/4
180°	4π/4	π
225°	5π/4	5π/4
270°	6π/4	3π/2
315°	7π/4	7π/4
360°	8π/4	2π

But why Radians? Well, it's because trig functions get messy when working with degrees. The 360° degrees was invented by the Babylonians as they used a base-60 system and 360° had a lot of factors. Meanwhile Radians are pure ratios between the arc and radius that, like degrees, are the same in every circle.

Now that we are familiar with radians, and how to convert them into degrees, let us look at another circle, that is known as the Unit Circle.

The Unit Circle in trigonometry has the radius of 1. Since we are dealing with ratios of numbers, the units don't matter, but we can take it as 1 unit. Since the circle has a radius of 1, the point where the circle intersects the positive x plane is (1,0), and the positive y plane at (0,1), etc.



Now when we take an angle θ, with its centre at the centre of the circle. Where θ intersects with the circumference, we draw a perpendicular line down to the x axis. What is formed is a right triangle with a hypotenuse of radius=1. This simplifies our trig functions down to —

$$\sin\theta = \text{Opp}/\text{Hyp} = \text{Opp}$$

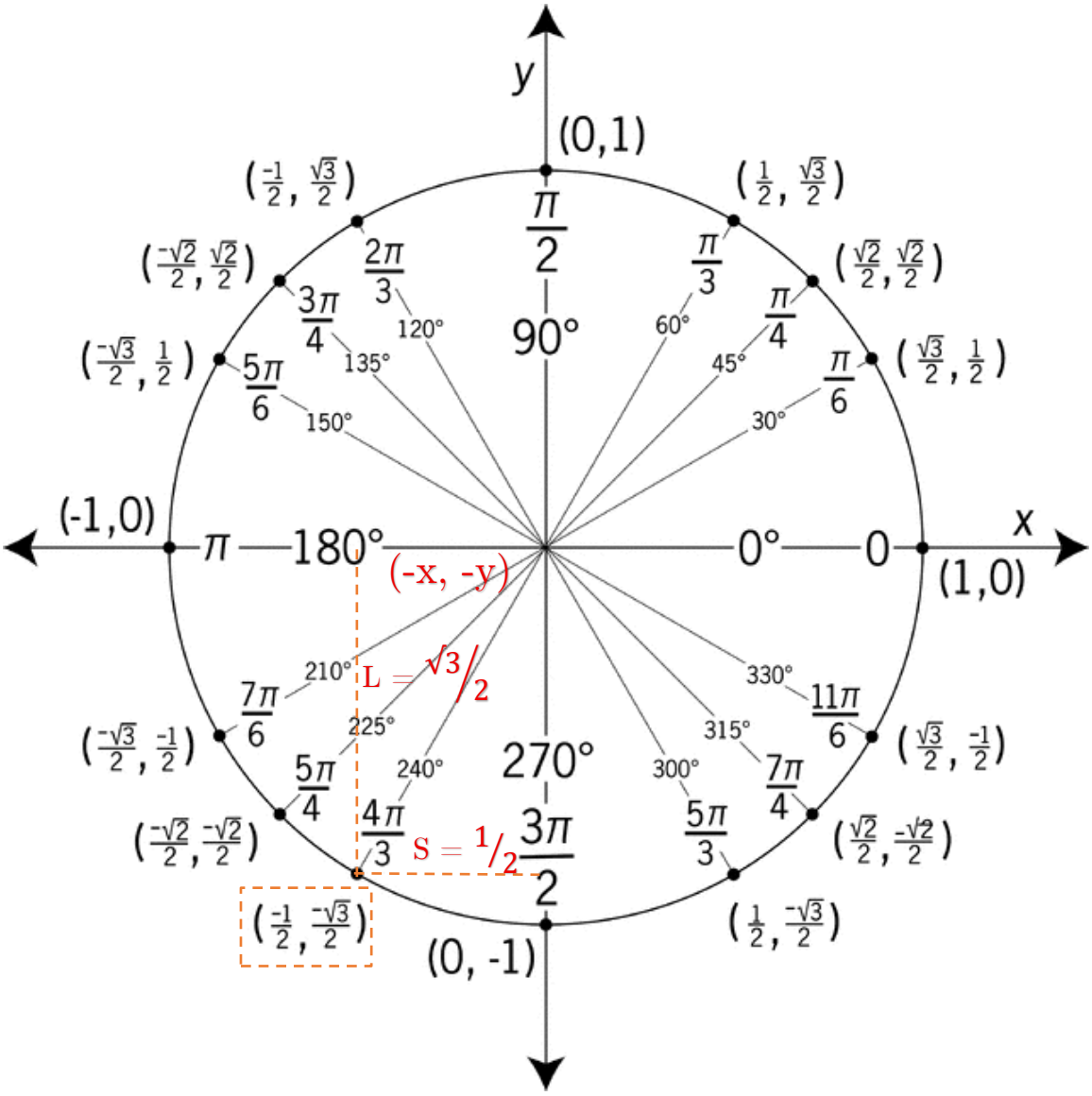
$$\cos\theta = \text{Adj}/\text{Hyp} = \text{Adj} \qquad\qquad \text{Hyp} = 1$$

$$\tan\theta = \text{Opp}/\text{Adj} = \sin\theta/\cos\theta$$

Which means, any angle θ, will have its intersecting point at (cosθ, sinθ) in the circumference. Remember this is only possible for a unit circle with radius of 1. For any other radii, multiply the radius with sinθ and cosθ so that it radius = r, (cosθ·r, sinθ·r)

Here we have shown the trig functions of sin and cos on the unit circle. We can also represent the other four functions however, that will be covered in the advanced trig analysis.

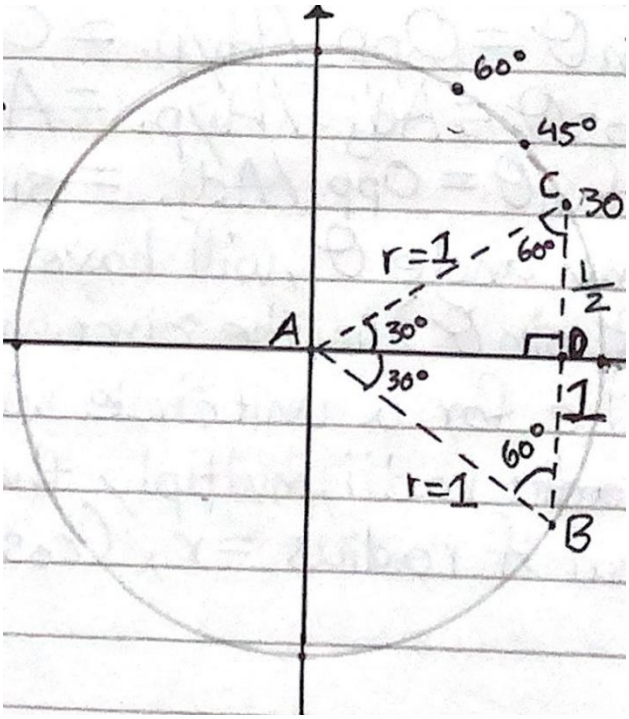
It is important to know the sin and cos (thereby the coordinates) of common angles, so here is the unit circle with degrees, radians and sin/cos, along with an aid for memorising them.



There are three numbers you should know. L is $\frac{\sqrt{3}}{2}$ M is $\frac{\sqrt{2}}{2}$ and S is $\frac{1}{2}$. You can remember it as $\sqrt{3}, \sqrt{2}$ and $\sqrt{1}$ (which is just 1), all over 2. Then the coordinates are just simply the distance of the point from the x or y axis, and the sign of x or y just depends on which quadrant it is in. For example, the coordinates of $4\pi/3$ (240°) are $(\frac{-1}{2}, \frac{-\sqrt{3}}{2})$ because the x value appears to be small so it equals S is $\frac{1}{2}$, while y value appears to be large so it equals L is $\frac{\sqrt{3}}{2}$, and in Quadrant IV we have, (-x,-y).

But why these numbers? Well here is the proof—

Proof that S= 1/2

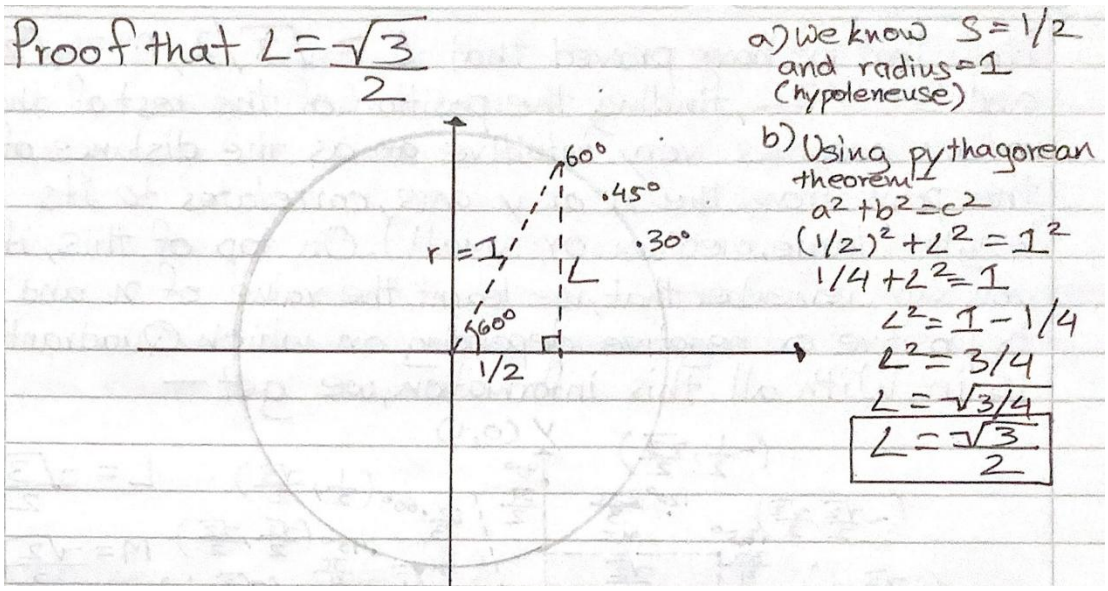


∠ACB = 60 because 30+90+60=180

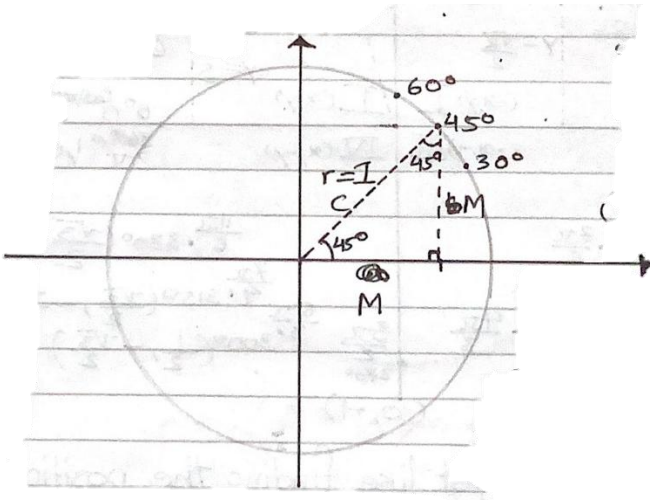
▲ABC is equilateral as all angles are the same therefore, all sides are the equal (to the radius,1)

CB = 1 so CD must be 1/2 or S = 1/2

Proof that L = √3/2



Proof that M = √2/2



Angles are 45°, 45°, and 90° due to angle sum property. Thus M = M

Pythagorean theorem—

$M^2+M^2=1^2$

$2M^2 = 1$

$M^2 = 1/2$

$M = \sqrt{1/2} = 1/\sqrt{2}$ or $\frac{1 \, x \, \sqrt{2}}{\sqrt{2} \, x \, \sqrt{2}} = \frac{\sqrt{2}}{2}$

And so, we conclude that these are all the ideas and concepts that someone getting into trigonometry would want to learn as an introduction into the matter. However, trigonometry is not limited to these topics, and further reading includes Law of Sines, Law of Cosines, Inverse trig functions, Polar and Complex Coordinates, Complex numbers, and finally going into Calculus.